Forecasting Armenian key macroeconomic indicators using factor-based dynamic models

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Abstract: Two model averaging approaches are used and compared in estimating and forecasting dynamic factor models, the well-known Bayesian model averaging (BMA) and the recently developed weighted average least squares (WALS). Both methods propose to combine frequentist estimators using Bayesian weights. We apply our framework to the Armenian economy using quarterly data from 2000–2010, and we estimate and forecast real GDP growth and inflation.

Keywords: Dynamic models, Factor analysis, Model averaging, Monte Carlo, Armenia

JEL Classifications: C11, C13, C52, C53, E52, E58

1 Introduction

In the recent macroeconomic literature, factor-based dynamic models have become popular. The idea underlying these models is that, while there are potentially a very large number of explanatory variables, most of the movement in the dependent variable can be explained by only a few variables or linear combinations thereof. One of the reasons why this happens it that the explanatory variables are often highly correlated.

We mention three recent examples where this approach has been successfully applied. Stock and Watson (2002) performed forecasting experiments for USA macroeconomic variables using 215 explanatory variables. From this large number of variables they extracted a few factors to forecast key

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macroeconomic indicators. Forni et al. (2000, 2003) provided a time-series forecasting method based on spectral analysis, and applied this method to forecast Euro-area industrial production and inflation using 447 explanatory variables. Finally, Bernanke et al. (2005) took a vector autoregressive (VAR) model and augmented it with factors based on 120 macroeconomic variables. All these papers find that the mean squared errors of estimates and forecasts based on factor models are lower than those obtained from vector autoregressive models.

After extracting factors, these models are typically estimated in the traditional econometric way, that is, separating model selection and estimation. Recent advances in econometric theory allow us to combine model selection and estimation into one procedure, thus avoiding the undesirable problem of pretesting. This procedure is called 'Bayesian model averaging'. The purpose of the current paper is to apply the basic (non-dynamic) model averaging framework to dynamics and factor extraction, and to use this dynamic framework to explain and forecast Armenian real GDP growth and inflation.

In addition, we wish to compare in this context the standard Bayesian model averaging (BMA) approach to the 'weighted average least squares' (WALS) approach, recently developed in Magnus et al. (2010). The WALS approach has both theoretical and computational advantages over BMA. Theoretical, because it generates bounded risk and contains an explicit treatment of ignorance; computational, because its computing time increases linearly rather than exponentially with the dimension of the model selection space. In Magnus et al. (2010), WALS was applied to growth empirics, but without dynamics or lagged dependent variables.

Estimation and forecasting in factor-based dynamic models using the BMA algorithm was first applied by Koop and Potter (2004) to US data. The current paper follows their general approach, but also reports on experiments where the two model averaging methods (WALS and BMA) are compared.

The paper is organized as follows. The factor-based dynamic model is introduced in Section 2. In Section 3 we present the WALS and BMA model averaging methods. In Section 4 are described macroeconomic time series dynamics. We report on two experiments. First, an estimation simulation in Section 5, then a forecast experiment in Section 6. Section 7 concludes.

2 The dynamic factor model

We consider the dynamic regression model

$$y_t = \alpha(L)y_{t-1} + \beta(L)x_{t-1} + \xi_t \qquad (t = 1, \ldots, T), \tag{1}$$

where y_t is a scalar dependent variable, x_t is a $k \times 1$ vector of nonrandom explanatory variables, $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator of dimensions p_1 and p_2, respectively, and ξ_t is a random vector of unobservable disturbances, independently and identically distributed as $\mathrm{N}(0, \sigma^2)$.

We have $p_1 + kp_2$ explanatory variables, which may be a large number. Moreover, many of the parameters may be close to zero. These two factors make it difficult to apply standard estimation methods (Koop and Potter, 2004). It is then common in the macro-econometric literature to replace the k explanatory variables with a much smaller number of variables. This can be achieved by using principal component or factor analysis algorithms. Then, after extracting the principal components, Model (1) can be rewritten as

$$y_t = \alpha(L)y_{t-1} + \gamma(L)f_{t-1} + \epsilon_t \qquad (t = 1, \dots, T), \tag{2}$$

where f_t $(m \times 1)$ is the vector of extracted principal components and $\gamma(L)$ is a polynomial in the lag operator (Stock and Watson, 2002). We assume that $m < k$ and $m < T$. Of course, as noted by Koop and Potter (2004, p. 553), there is a cost in this type of transformation, namely that the interpretation of the variables is more difficult.

Koop and Potter (2004) were the first to show how Bayesian model averaging can be applied to estimation and forecasting using dynamic factor models. Their study applies BMA to the problem of forecasting GDP growth and inflation using quarterly US data on 162 time series. This paper follows their approach, but also compares two competing estimation procedures: BMA and WALS. This will not only tell us something about the power of the two algorithms, but will also provide information about the robustness of our results.

3 Estimation methodology

The idea behind combining estimators (or forecasts) is to use information from all models within a given family in a continuous fashion. In contrast to standard econometrics — where one first selects a model and then estimates the parameters within the chosen model, a discrete procedure — we combine the estimates from all models considered, where some models get a higher weight than others, based on priors and diagnostics. One advantage of this procedure is that we avoid the well-known pretest problem: our procedure is a joint procedure, where model selection and estimation are combined.

Thus, we choose the linear regression model

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon = X\beta + \epsilon, \qquad \epsilon \sim \mathrm{N}(0, \sigma^2 I_n),$$

where y $(n \times 1)$ is the vector of observations, X_1 $(n \times k_1)$ and X_2 $(n \times k_2)$ are matrices of nonrandom regressors, ϵ is a random vector of unobservable disturbances, and β_1 and β_2 are unknown parameters which we need to estimate. We assume that $k_1 \geq 1$, $k_2 \geq 0$, $k = k_1 + k_2 \leq n - 1$, that $X = (X_1 : X_2)$ has full column-rank, and that the disturbances are independent and identically distributed.

The reason for distinguishing between X_1 and X_2 is that X_1 contains variables that we want to be in the model (whatever t-values or other diagnostics we find), while X_2 contains variables that may or may not be in the model. The columns of X_1 are called 'focus' regressors, the columns of X_2 'auxiliary' regressors. The uncertainty about each auxiliary regressor, that is whether we should or should not include the regressor in our model, is a very common situation, and the application of model averaging is then a natural procedure. Rather than choosing one model by preliminary diagnostic tests, we assume that each model tells us something of interest about our focus parameters. We do not, however, trust each model to the same degree, and the resulting weights are determined by priors and data. In this paper we concentrate on two model averaging algorithms, the well-known BMA algorithm and the recently introduced WALS algorithm. We briefly summarize each in turn. Full details are provided in Magnus et al. (2010).

Bayesian model averaging (BMA). With the exception of Magnus et al. (2010), the whole literature on Bayesian model averaging considers the case $k_1 = 1$. We summarize the approach of Magnus et al. (2010, Section 2). Since there are k_2 auxiliary regressors, we have 2^{k_2} different models to consider, because each auxiliary regressor can either be included or not. For each subset X_{2i} of $k_{2i} \leq k_2$ auxiliary variables we consider the regression

$$y = X_1\beta_1 + X_{2i}\beta_{2i} + \epsilon_i,$$

which we call model $\mathcal{M}_i$. If we let $p(\mathcal{M}_i)$ denote the prior probability that $\mathcal{M}_i$ is the true model, then the posterior probability for model $\mathcal{M}_i$ is given by

$$\lambda_i = p(\mathcal{M}_i|y) = \frac{p(\mathcal{M}_i)p(y|\mathcal{M}_i)}{\sum_j p(\mathcal{M}_j)p(y|\mathcal{M}_j)} \qquad (i = 1, \ldots, 2^{k_2}),$$

and if we take $p(\mathcal{M}_i) = 2^{-k_2}$, which is the common assumption, then $p(\mathcal{M}_i)$ does not depend on i, and we have simply $\lambda_i \propto p(y|\mathcal{M}_i)$, the marginal likelihood of y in model $\mathcal{M}_i$. If we adopt Zellner's g-prior, then

$$\lambda_i \propto \left(\frac{g_i}{1+g_i}\right)^{k_{2i}/2} (y'M_1A_iM_1y)^{-(n-k_1)/2},$$

where

$$A_i = \frac{g_i}{1+g_i} M_1 + \frac{1}{1+g_i}(M_1 - M_1 X_{2i}(X_{2i}' M_1 X_{2i})^{-1} X_{2i}' M_1)$$

and

$$M_1 = I_n - X_1(X_1' X_1)^{-1} X_1'.$$

We specify g_i as

$$g_i = \frac{1}{\max(n, k_2^2)}.$$

The λ_i are the required weights to obtain the BMA estimates and precisions. For example, the BMA estimator of β_1 is given by

$$\mathrm{E}(\beta_1|y) = \sum_{i=1}^{k_2} \lambda_i \, \mathrm{E}(\beta_1|y, \mathcal{M}_i).$$

There are several problems with BMA. First, all 2^{k_2} models have to be evaluated implying a huge computational effort; second, the priors are based on the normal distribution, leading to unbounded risk; and third, the treatment of 'ignorance' is ad hoc and unsatisfactory. These problems are avoided in an alternative model averaging procedure, called WALS.

Weighted average least squares (WALS). In the WALS algorithm, developed in Magnus et al. (2010, Section 3), we first 'orthogonalize' the columns of X_2 such that $P'X_2'M_1X_2P = \Lambda$, where P is orthogonal and Λ is diagonal. Then we define $X_2^* = X_2 P \Lambda^{-1/2}$ and $\beta_2^* = \Lambda^{1/2} P' \beta_2$, so that $X_2^* \beta_2^* = X_2 \beta_2$. Our prior will be on β_1 and β_2^* (rather than on β_2), and this gives us enormous computational advantage, because all models which include x_{2j}^* as a regressor will have the same estimator of β_{2j}^*, irrespective which other β_2^*'s are estimated.

The second ingredient is the 'equivalence theorem' (Magnus and Durbin, 1999; Danilov and Magnus, 2004), which tells us that the WALS estimator b_1 of β_1 will be 'good' (in the mean squared error sense) if and only if $W\hat{\beta}_2^*$ is a good estimator of β_2^*, where $\hat{\beta}_2^*$ denotes the least squares estimator of β_2^* in the unrestricted model, and W is a random diagonal matrix of order $k_2 \times k_2$. The diagonal elements w_j of W will depend on the weights λ_i, but while there are 2^{k_2} λ's, there are only k_2 w's. This is where the computational advantage comes from.

The third ingredient is the treatment of ignorance. Based on the fact that a t-value of one indicates that including an auxiliary regressor gives us the same mean squared error of the estimated focus parameter as excluding the auxiliary regressor, we define ignorance on an auxiliary parameter η by the properties

$$\Pr(\eta > 0) = \Pr(\eta < 0), \qquad \Pr(|\eta| > 1) = \Pr(|\eta| < 1),$$

and we propose the Laplace distribution

$$\pi(\eta) = (c/2)\exp(-c|\eta|)$$

with $c = \log 2$.

The WALS estimator is a Bayesian combination of frequentist estimators, and possesses major advantages over standard Bayesian model averaging (BMA) estimators: the WALS estimator has bounded risk, allows a coherent treatment of ignorance, and its computational effort is negligible. The sampling properties of the WALS estimator as compared to BMA estimators have been examined in Magnus et al. (2011), where Monte Carlo evidence shows that the WALS estimator performs better than standard BMA and pretest alternatives. Because of the light computational cost, extensions are possible in many directions. For example, Magnus et al. (2011) extend the WALS theory to allow for nonspherical disturbances.

In the current paper we consider a broader class of linear models than before, by allowing the regressors to include lagged dependent variables. The y_t will then be correlated with the current and all previous disturbances, but uncorrelated with all future disturbances. Hence, the regressor y_{t-1} will be uncorrelated with the current disturbance and all future disturbances, although it will be correlated with all previous disturbances. The standard ordinary least squares (OLS) assumptions do therefore not hold, and the finite-sample properties of the least squares estimators are not valid. However, as shown by Mann and Wald (1943), these properties will hold asymptotically.

We need to determine which variables are focus and which are auxiliary. The focus variables are those which we want in the model on theoretical or other grounds, irrespective of any diagnostics. The choice is not always easy and often subjective. It is guided by economic-theoretical considerations and by previous empirical experience. But it is also guided by the purpose of the model: if our primary purpose is to study the effect of x and z on y, then it would seem ill-advised to remove x or z from the model; these are necessarily focus variables.

In our setting, we shall assume that the lagged dependent variables are always focus regressors. But the extracted principal components can be either focus or auxiliary. Thus we write

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon, \tag{3}$$

where X_1 contains the lagged dependent variables and a subset (possible empty) of the principal components, and X_2 contains the remainder of the principal components. In this form we can apply BMA and WALS to this system.

4 Data description and preliminary analysis

Our data consist of quarterly time series of 42 macroeconomic variables from 2000 (second quarter) to 2010 (fourth quarter), in total 43 observations for each variable. This set comprises information on national accounts data (9 variables) and consumer prices and exchange rate data (13 variables), listed in Table 1; and on financial and monetary policy indicators (13 variables) and international macroeconomic indicators (7 variables), listed in Table 2. All variables in Table 1 are in logarithmic form, in first differences. The variables in column 1 are all real. The variables in columns 1 and 2 are seasonally adjusted.

Table 1: National accounts, consumer prices, and exchange rates

National accounts	Price indices	Price indices and exchange rates
GDP	Consumer price index	Wheat price index
Consumption	Food price index	Fuel price index
Investment	Nonfood price index	Imported food price index
Exports	Services price index	Imported nonfood price index
Imports	Home food price index	Administrative price index
Industrial output		AMD/USD exchange rate
Agricultural output		AMD/EURO exchange rate
Construction		AMD/RR exchange rate
Services		

The variables in Table 2 are also in logarithmic form, in first differences, and the variables in columns 1 and 3 are seasonally adjusted. The international indicators in column 3 are taken from the International Financial Statistics (IFS) published by the IMF and are already seasonally adjusted.

In this paper we estimate and forecast factor-based dynamic models using principal components. These principal components are based on the underlying data set of 40 variables (excluding dependent variables, that is real GDP growth and inflation). The extracted principal components have been given names, based on the correlation coefficients between the extracted principal components and the underlying time series. Some important characteristics

Table 2: Financial, monetary, and international indicators

Financial policy indicators	Interest rates	International indicators
Cash money	AMD deposits	USA real GDP
Money aggregate, M0	USD deposits	EU real GDP
Money aggregate, M1	AMD loans	USA consumer price index
Money aggregate, M2X	USD loans	EU consumer price index
Total deposits	Central Bank interbank	Gasoline price index
Loans to economy		Petroleum price index
Loans to enterprizes		Wheat price index
Loans to households		

Table 3: Characteristics of the extracted principal components

Principal components	Rotated eigenvalue	% of total variance	Cumulative %	Correlation with growth	Correlation with inflation
Int_rate	5.11	12.78	12.78	0.04	−0.21
Ex_rate	5.00	12.51	25.29	−0.03	0.28
Invest	3.90	9.74	35.03	0.65	0.07
Mon_agg	3.60	9.00	44.03	0.43	0.01
Credit	3.19	7.98	52.01	0.02	0.10
Pr_index	2.62	6.54	58.55	0.23	0.62
ImpExp	2.58	6.46	65.01	0.22	−0.03
Nat_acc	2.37	5.93	70.93	0.31	−0.27
Gstar	2.01	5.01	75.95	0.29	−0.12
Hfood_pr	1.69	4.23	80.18	0.09	0.47

of the extracted principal components are presented in the Table 3. The first principal component is *Int_rate* and its contribution to the total variance of the underlying variables is 12.78%. The second principal component is *Ex_rate* with a contribution of 12.51%, and the third is *Invest* with a contribution of 9.74%. The ten most important principal components (those with a rotated eigenvalue larger than 1) explain more than 80% of the variance of the underlying variables, which we consider to be sufficient.

Each of the extracted principal components could be used for estimation in our factor-based dynamic models. However, we use our knowledge of economic theory and Armenian practice to include only those principal components which contain important information about real GDP growth and inflation. Regarding real GDP growth, the highest correlations are obtained by *Invest*, *Mon_agg*, *Pr_index*, *ImpExp*, *Nat_acc*, and *Gstar*. Regarding inflation, the highest correlations are obtained by *Int_rate*, *Ex_rate*, *Credit*,

Table 4: Focus and auxiliary variables ($j = 1, \ldots, 4$)

Growth G			Inflation INF		
Regressor	Model 1.1	Model 1.2	Regressor	Model 2.1	Model 2.2
Intercept	focus	focus	*Intercept*	focus	focus
G_{t-j}	focus	focus	INF_{t-j}	focus	focus
$Invest_{t-j}$	auxiliary	focus	Ex_rate_{t-j}	auxiliary	focus
$ImpExp_{t-j}$	auxiliary	focus	Pr_index_{t-j}	auxiliary	focus
Nat_acc_{t-j}	auxiliary	focus	$Hfood\_pr_{t-j}$	auxiliary	focus
Mon_agg_{t-j}	auxiliary	auxiliary	Int_rate_{t-j}	auxiliary	auxiliary
Pr_index_{t-j}	auxiliary	auxiliary	$Credit_{t-j}$	auxiliary	auxiliary
$Gstar_{t-j}$	auxiliary	auxiliary	Nat_acc_{t-j}	auxiliary	auxiliary
			$Gstar_{t-j}$	auxiliary	auxiliary

Pr_index, *Nat_acc*, *Gstar*, and *Hfood_pr*.

These choices then lead to the four models in Table 4. Model 1 refers to GDP growth and Model 2 to inflation. The dependent variables are either 'growth', denoted G, defined as the quarterly growth rate of real GDP, and 'inflation', denoted *INF*, defined as the quarterly growth rate of the consumer price index CPI. As we can see from table each model has two variants. In variant 1 (Models 1.1 and 2.1) we take as our focus variables only the lagged values of the dependent variable (and the intercept), while all other variables are auxiliary, that is, we are uncertain whether they should be in the model or not. This is the same type of specification as in Koop and Potter (2004). In variant 2 (Models 1.2 and 2.2) we have more focus variables. Here we argue that some of the extracted principal components must always be in the model so that they should be treated as focus variables. For Model 1.2 this applies to *Invest*, *ImpExp*, and *Nat_acc*, because the level of real GDP growth depends directly on the level of these components. For Model 2.2 it applies to *Ex_rate*, *Pr_index*, and *Hfood_pr*, because these principal components are known to have a direct impact on the rate of inflation. Having thus specified the four factor-based dynamic models, we now turn to their estimation and forecasting simulation experiments.

5 An estimation simulation experiment

In this part we conduct a comparison between BMA and WALS estimation results using Monte-Carlo simulations. For that we assume that we know the true data-generating process (DGP) and can therefore compare the estimates to the truth. We conduct the simulation experiments for one, two, and three lags, so that we gain insight on the performance of the WALS and BMA algorithms for various lag lengths. In Tables 5 and 6 we present the parameter values in the data-generating processes for the growth and inflation models, respectively.

Table 5: Data-generation process, Model 1 (Growth), Version 2

	One lag	Two lags	Three lags
Intercept	0.50	1.20	3.00
G_{t-1}	0.75	0.95	0.80
$Invest_{t-1}$	−0.30	−0.70	−0.40
$ImpExp_{t-1}$	−0.15	0.00	0.10
Nat_acc_{t-1}	0.60	0.15	0.55
Mon_agg_{t-1}	0.30	0.40	0.40
Pr_index_{t-1}	0.20	0.35	0.30
$Gstar_{t-1}$	−0.35	0.10	−0.30
G_{t-2}	—	−0.60	−1.30
$Invest_{t-2}$	—	0.30	1.30
$ImpExp_{t-2}$	—	0.30	0.40
Nat_acc_{t-2}	—	0.95	1.15
Mon_agg_{t-2}	—	0.40	0.70
Pr_index_{t-2}	—	−0.25	0.90
$Gstar_{t-2}$	—	0.05	0.90
G_{t-3}	—	—	−0.10
$Invest_{t-3}$	—	—	0.75
$ImpExp_{t-3}$	—	—	0.45
Nat_acc_{t-3}	—	—	0.60
Mon_agg_{t-3}	—	—	0.90
Pr_index_{t-3}	—	—	−0.30
$Gstar_{t-3}$	—	—	−0.30
σ^2	2.25	2.25	2.25

We randomly draw the $\{u_t\}$ from a standard-normal distribution. Then, given the data-generating process and the values of the regressors, we generate the time series for real GDP growth or inflation, the dependent variables. Now that we have all the data, we estimate the parameters using the models and the BMA and WALS estimation algorithms. This gives us parameter estimates. Next we draw new errors $\{u_t\}$, obtain new values for the dependent variable, and hence new parameter estimates. We repeat this 1000 times,

Table 6: Data-generation process for Model 2 (Inflation), Version 2

	One lag	Two lags	Three lags
Intercept	1.00	0.10	−2.00
INF_{t-1}	0.10	0.80	1.40
Ex_rate_{t-1}	0.20	0.60	−0.10
Pr_index_{t-1}	0.55	−0.15	−0.15
$Hfood\_pr_{t-1}$	0.10	−0.15	−0.85
Int_rate_{t-1}	−0.20	0.00	0.50
$Credit_{t-1}$	0.10	−0.55	−0.50
Nat_acc_{t-1}	−0.10	−0.70	0.70
$Gstar_{t-1}$	−0.30	−0.40	0.40
INF_{t-2}	—	0.50	1.00
Ex_rate_{t-2}	—	−0.50	−0.15
Pr_index_{t-2}	—	−0.50	−0.75
$Hfood\_pr_{t-2}$	—	0.40	−0.50
Int_rate_{t-2}	—	0.50	0.60
$Credit_{t-2}$	—	0.40	0.60
Nat_acc_{t-2}	—	0.40	0.80
$Gstar_{t-2}$	—	0.40	0.40
INF_{t-3}	—	—	0.30
Ex_rate_{t-3}	—	—	0.25
Pr_index_{t-3}	—	—	−0.65
$Hfood\_pr_{t-3}$	—	—	−0.20
Int_rate_{t-3}	—	—	−0.50
$Credit_{t-3}$	—	—	0.60
Nat_acc_{t-3}	—	—	−0.50
$Gstar_{t-3}$	—	—	0.50
σ^2	1.44	1.44	1.44

and compute the simulation root mean squared errors (RMSE):

$$\text{RMSE}_k^{wals} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{wals_l} - \beta_k^{true})^2},$$

$$\text{RMSE}_k^{bma} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{bma_l} - \beta_k^{true})^2},$$

where β_k^{true} denotes the true value of β_k, and $\beta_k^{wals_l}$ and $\beta_k^{bma_l}$ are the corresponding WALS and BMA estimates, respectively, for the l-th iteration.

Table 7: RMSE for estimation simulations, Model 1 (Growth), Version 2

	WALS	BMA	WALS	BMA	WALS	BMA
Intercept	0.0373	0.0375	0.0293	0.0270	0.0330	0.0399
G_{t-1}	0.0192	0.0193	0.0292	0.0283	0.0247	0.0222
$Invest_{t-1}$	0.0340	0.0341	0.0366	0.0362	0.0212	0.0216
$ImpExp_{t-1}$	0.0129	0.0130	0.0116	0.0118	0.0101	0.0099
Nat_acc_{t-1}	0.0172	0.0173	0.0190	0.0199	0.0132	0.0122
Mon_agg_{t-1}	0.0164	0.0206	0.0142	0.0151	0.0134	0.0180
Pr_index_{t-1}	0.0099	0.0110	0.0100	0.0124	0.0104	0.0119
$Gstar_{t-1}$	0.0154	0.0126	0.0132	0.0064	0.0202	0.0148
G_{t-2}	—	—	0.0163	0.0170	0.0380	0.0395
$Invest_{t-2}$	—	—	0.0146	0.0141	0.0480	0.0516
$ImpExp_{t-2}$	—	—	0.0092	0.0099	0.0113	0.0119
Nat_acc_{t-2}	—	—	0.0105	0.0109	0.0161	0.0171
Mon_agg_{t-2}	—	—	0.0082	0.0098	0.0150	0.0194
Pr_index_{t-2}	—	—	0.0097	0.0088	0.0259	0.0259
$Gstar_{t-2}$	—	—	0.0124	0.0065	0.0254	0.0288
G_{t-3}	—	—	—	—	0.0056	0.0053
$Invest_{t-3}$	—	—	—	—	0.0263	0.0244
$ImpExp_{t-3}$	—	—	—	—	0.0124	0.0134
Nat_acc_{t-3}	—	—	—	—	0.0138	0.0140
Mon_agg_{t-3}	—	—	—	—	0.0161	0.0202
Pr_index_{t-3}	—	—	—	—	0.0104	0.0087
$Gstar_{t-3}$	—	—	—	—	0.0109	0.0088

The results of the Monte-Carlo simulations are presented in Tables 7 (for growth) and 8 (for inflation). The main purpose of these simulations is to compare BMA and WALS. WALS has certain theoretical and computational advantages, but does it in fact perform better than BMA? The simulations suggest that this might be the case, although the difference is small. In the growth simulations, WALS achieves a lower RMSE than BMA for 88% (one lag), 53% (two lags), and 61% (three lags) of the parameters, thus outperforming BMA. In the inflation simulations, the percentages are somewhat lower: 39% (one lag), 59% (two lags), and 48% (three lags). Hence, a slight advantage for WALS over BMA.

The above estimation simulations were based on the assumption that the data-generation process and the model coincide. For example, if the DGP has one lag, then we use a model with one lag. This, of course, is not realistic, since in practice we don't know the DGP and therefore the chance that our

Table 8: RMSE for estimation simulations, Model 2 (Inflation), Version 2

	WALS	BMA	WALS	BMA	WALS	BMA
Intercept	0.0095	0.0089	0.0529	0.0538	0.0882	0.0910
INF_{t-1}	0.0067	0.0062	0.0261	0.0257	0.0409	0.0416
Ex_rate_{t-1}	0.0061	0.0061	0.0126	0.0124	0.0174	0.0180
Pr_index_{t-1}	0.0065	0.0065	0.0161	0.0163	0.0269	0.0252
$Hfood\_pr_{t-1}$	0.0067	0.0067	0.0200	0.0216	0.0341	0.0354
Int_rate_{t-1}	0.0052	0.0057	0.0197	0.0184	0.0455	0.0253
$Credit_{t-1}$	0.0047	0.0038	0.0138	0.0163	0.0210	0.0202
Nat_acc_{t-1}	0.0046	0.0038	0.0117	0.0138	0.0198	0.0209
$Gstar_{t-1}$	0.0052	0.0070	0.0101	0.0127	0.0171	0.0123
INF_{t-2}	—	—	0.0195	0.0204	0.0341	0.0351
Ex_rate_{t-2}	—	—	0.0093	0.0073	0.0127	0.0107
Pr_index_{t-2}	—	—	0.0189	0.0187	0.0297	0.0318
$Hfood\_pr_{t-2}$	—	—	0.0127	0.0120	0.0210	0.0215
Int_rate_{t-2}	—	—	0.0169	0.0135	0.0318	0.0252
$Credit_{t-2}$	—	—	0.0115	0.0127	0.0164	0.0132
Nat_acc_{t-2}	—	—	0.0089	0.0092	0.0185	0.0224
$Gstar_{t-2}$	—	—	0.0064	0.0092	0.0120	0.0108
INF_{t-3}	—	—	—	—	0.0111	0.0105
Ex_rate_{t-3}	—	—	—	—	0.0117	0.0084
Pr_index_{t-3}	—	—	—	—	0.0134	0.0108
$Hfood\_pr_{t-3}$	—	—	—	—	0.0115	0.0115
Int_rate_{t-3}	—	—	—	—	0.0289	0.0176
$Credit_{t-3}$	—	—	—	—	0.0161	0.0161
Nat_acc_{t-3}	—	—	—	—	0.0084	0.0124
$Gstar_{t-3}$	—	—	—	—	0.0097	0.0106

chosen model happens to be the DGP is negligible. We now consider one case where the model is underspecified. More specifically, the DGP has three lags, but the model has only one lag. We estimate the parameters in the one-lag model and compare with the corresponding (true) parameters in the three-lag DGP. The results are presented in Table 9. Here, also, WALS appears to be at an advantage. For 88% (growth) and 61% (inflation) of the parameters, WALS achieves a lower RMSE than BMA.

Table 9: RMSE for estimation simulations in the case of misspecification, Models 1 and 2, Version 2

Growth			Inflation		
	WALS	BMA		WALS	BMA
Intercept	0.0297	0.0302	*Intercept*	0.0520	0.0513
G_{t-1}	0.0201	0.0202	INF_{t-1}	0.0442	0.0437
$Invest_{t-1}$	0.0356	0.0357	Ex_rate_{t-1}	0.0154	0.0153
$ImpExp_{t-1}$	0.0131	0.0132	Pr_index_{t-1}	0.0297	0.0297
Nat_acc_{t-1}	0.0181	0.0182	$Hfood\_pr_{t-1}$	0.0263	0.0264
Mon_agg_{t-1}	0.0175	0.0223	Int_rate_{t-1}	0.0107	0.0127
Pr_index_{t-1}	0.0103	0.0121	$Credit_{t-1}$	0.0101	0.0123
$Gstar_{t-1}$	0.0162	0.0121	Nat_acc_{t-1}	0.0167	0.0188
			$Gstar_{t-1}$	0.0107	0.0115

6 A forecast experiment

We conduct a second experiment, this time in forecasting rather than estimation. Suppose we use $T_1 < T = 42$ quarters on which we base our estimates. This leaves us $T_2 = T - T_1 > 0$ quarters for forecast experiments. The h-period forecast is given by

$$\hat{y}_{T_1+h} = \hat{\alpha}(L)y_{T_1+h-1} + \hat{\gamma}(L)f_{T_1+h-1} \qquad (h = 1, \ldots, T_2),$$

where y denotes either GDP growth or inflation. In a practical situation we would not know f_{T_1+h-1} and y_{T_1+h-1}, when $h \geq 2$. So we would have to forecast these as well. In the experiment we use the observed values of f_{T_1+h-1} and y_{T_1+h-1}, hence not the forecasted value $\hat{y}_{T_1+h-1}$ when $h \geq 2$. Then we compute

$$\text{RMSE}_{T_1} = \sqrt{\frac{1}{T - T_1} \sum_{h=1}^{T-T_1} (\hat{y}_{T_1+h} - y_{T_1+h})^2},$$

which depends on the estimation period T_1, the model, and the method (BMA or WALS). The results are presented in Tables 10 and 11 .

In this case we have calculated the RMSE not only for BMA and WALS, but also for two traditional methods of estimation: general-to-specific (GtS) model selection followed by estimation of the selected model, and ordinary least squares (OLS) of the unrestricted model. Including these standard forecasting methods allows us to compare model averaging with more traditional methods.

Table 10: RMSE for ex-post forecast accuracy, Model 1 (Growth)

			T_1				
Number of lags	Version	Method	38	37	36	35	34
One lag	1	WALS	0.8557	0.9967	2.5503	6.1158	3.6533
		BMA	0.8338	0.9726	2.4993	6.3675	3.5549
		GtS	0.9606	1.1124	2.6739	6.0492	3.4711
		OLS	0.9847	1.0726	3.2020	5.6782	3.6570
	2	WALS	0.8597	1.0181	2.8576	5.8070	3.6330
		BMA	0.9416	1.1265	2.5818	5.9392	3.6009
		GtS	1.1311	1.2979	2.3720	6.0667	3.5998
		OLS	0.9847	1.0726	3.2020	5.6782	3.6570
Two lags	1	WALS	2.2203	2.8415	3.6176	2.7037	2.6341
		BMA	1.8333	2.3204	3.2558	2.4816	1.7849
		GtS	2.0147	2.9072	3.1916	2.4471	1.5800
		OLS	2.6610	3.3104	3.5279	3.2889	3.2062
	2	WALS	2.2162	2.8118	3.5271	3.1048	2.7043
		BMA	2.2155	2.6711	3.6429	3.0343	2.5689
		GtS	2.9367	3.4904	3.7139	3.1051	2.8986
		OLS	2.6610	3.3104	3.5279	3.2889	3.2062
Three lags	1	WALS	2.3276	2.5872	4.3087	4.5578	2.8051
		BMA	2.1199	1.9616	3.2988	3.8783	3.0844
		GtS	2.0535	2.5983	4.1221	4.5073	3.5460
		OLS	2.5757	3.1871	4.4612	4.9230	3.1832
	2	WALS	2.2043	2.7098	4.1208	4.6082	3.2474
		BMA	2.1169	2.8038	4.2715	4.6699	3.6258
		GtS	2.8060	1.5199	3.8202	4.4603	4.3567
		OLS	2.5757	3.1871	4.4612	4.9230	3.1832

For all cases, the smaller is the estimation period T_1, the less accurate are the estimates and the forecasts, that is, the RMSE increases as T_1 decreases. This is to be expected and it happens most of the time, but not always. In particular the behavior for $T_1 = 35$ is different. The explanation lies in the global financial crisis, which affected Armenia heavily. From the third quarter of 2008 (quarter 34 in our data set) to the second quarter of 2009

Table 11: RMSE for ex-post forecast accuracy, Model 2 (Inflation)

			T_1				
Number of lags	Version	Method	38	37	36	35	34
One lag	1	WALS	0.8542	0.7807	0.9050	0.8545	0.8993
		BMA	0.8851	0.8810	0.9949	0.9373	0.9697
		GtS	0.9906	1.0484	1.0819	1.0232	1.0198
		OLS	0.9060	0.8448	0.8741	0.8788	0.8842
	2	WALS	0.8923	0.8061	0.9000	0.8813	0.8865
		BMA	0.9579	0.8718	0.9787	0.9291	0.9421
		GtS	1.0024	0.9252	1.0051	0.9481	0.9559
		OLS	0.9060	0.8448	0.8741	0.8788	0.8842
Two lags	1	WALS	1.6452	1.6568	1.5987	1.7970	1.5262
		BMA	1.0726	0.9829	0.8997	1.0536	0.9385
		GtS	1.0536	0.9371	0.7445	1.0959	0.8920
		OLS	2.0139	2.1006	2.0722	2.3070	1.8852
	2	WALS	1.6357	1.6463	1.5935	1.7557	1.5967
		BMA	1.1079	1.0094	1.0292	1.1293	1.3293
		GtS	1.0076	0.9035	1.0614	1.1510	1.2761
		OLS	2.0139	2.1006	2.0722	2.3070	1.8852
Three lags	1	WALS	4.5016	3.8276	4.1335	3.9527	2.7138
		BMA	1.2269	1.1040	1.0326	0.9662	1.1329
		GtS	6.1520	1.1017	4.7159	4.7402	4.5196
		OLS	6.1409	5.2268	5.7689	5.4534	3.5626
	2	WALS	4.2447	3.4789	3.9977	3.8076	2.4278
		BMA	1.3646	1.2628	1.3900	1.2499	1.9155
		GtS	0.9806	1.0851	2.0882	1.9551	1.7758
		OLS	6.1409	5.2268	5.7689	5.4534	3.5626

(quarter 37) Armenia's GDP decreased by 18%. The largest decrease (around 9.0%) in real GDP took place in the fourth quarter of 2008 (quarter 35). Such a large decrease in real GDP causes a large deviation of real GDP from its long-term trend, and this explains (in part) why the RMSE values calculated for $T_1 = 35$ are relatively large, and for $T_1 = 36$ somewhat smaller.

Two main conclusions emerge from Tables 10 and 11. First, we see that the model averaging techniques WALS and BMA outperform the more traditional methods GtS and OLS. But the choice between WALS and BMA is still ambiguous. While in the estimation simulations we found that WALS performs better than BMA, we find in the forecasting simulations that BMA performs better than WALS in 2/3 of the 30 forecasts, both for growth and for inflation.

7 Concluding remarks

We have applied two alternative model averaging algorithms (WALS and BMA) to the problem of estimating factor-based dynamic models in Armenia. The same models are also used to forecast two key macroeconomic variables, namely real GDP growth and inflation. The theoretical advantage of using model averaging is that it allows all models to play a role in the estimation and forecasting, thus avoiding the problem of pretesting. A comparison of the WALS to the BMA algorithm does not reveal large differences in performance. The WALS methodology has a stronger theoretical appeal, but — in the current context — there is not sufficient evidence to prefer one over the other. The simulations do show, however, that both model averaging methods outperform the more traditional methods (general-to-specific and OLS).

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