YIELD CURVE ESTIMATION AND FORECASTING IN ARMENIA*

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Abstract – Modeling the yield curve dynamics is very important for investors, practioners from the central banks and market analyst in general. In this paper we used the Diebold-Li interpretation to the Nelson-Siegel model in order to fit and forecast the Armenian yield curve dynamics. The data employed consist of monthly government bonds interest rates from January 2005 to December 2017. We rewrite the Nelson-Siegel (1987) model in the form of Diebold and Li (2006) and estimate the unknown parameters by using Two-step and State-space Kalman filter approaches. The results obtained for the out of sample ex post forecasts showed that the State-space Kalman filter is the better approach. This is because first it allows simultaneously to estimate all unknown parameters in the model and second it has abilities to forecast yield curve more accurately when compared with another alternative models, such as Random walk and Two-step. Although these differences are not statistically significant when we apply Diebold-Mariano test statistics.

Keywords**:** Yield curve, Nelson-Siegel model, Armenian financial market, State-space model, Kalman filter, Diebold-Li model, government bonds yield.
JEL Classification: G12, E43, E44, E58

1. INTRODUCTION

In normal market conditions, yield curves are usually upward sloping asymptotically: the longer the maturity, the higher the yield, with diminishing marginal increases (that is, as one moves to the right, the curve flattens out). This can be explained by the liquidity premium theory, which suggests that investors demand higher premiums as maturities rise since liquidity is sacrificed for longer. This premium compensates investors for the added risk of having their money tied up for a longer period, including the greater price uncertainty. Because of the term premium, long-term bond yields tend to be higher than short-term yields and the yield curve slopes upward.

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However, the more widely accepted notion for the upward sloping yield curve follows the market expectation hypothesis, which suggests that yields on longer-term bonds are formed by the current and expected future short-term rates. Thus, an expected increase in future short-term rates would cause an upward sloping yield curve. In contrast to the normal curve, an inverted yield curve is downward sloping and signals expectations of lower short-term rates in the future compared to the present. However, as we move further the yield curve, the market expectations hypothesis weakens since the market cannot anticipate interest rate changes far in time and only the liquidity premium theory remains, which explains the flattening of the long end of the normal yield curve.

Plotting the yield to visualize the term structure of interest rates is important to macroeconomists, financial economists, policymakers, and portfolio managers as it indicates investors and analysts expectations. For instance, the normal curve indicates the expected increase in short-term rates in the future, which is an indicator of expected economic growth and/or inflation. An inverted curve is considered to be a good indicator of a recession and possible deflation, that will cause central banks to lower interest rates in order to simulate the economy and prevent deflation. Thus, as a significant portion of the yield curve is formed by market expectations of short- and long-term central bank policy, it is used by policymakers to understand market expectations and evaluate the perceived central bank credibility or identify any market overreactiin or misunderstanding of policy intentions.

In this paper, we want to consider the applicability of the advanced methodology for estimation and forecasting the government bond yields dynamics to developing economies like Armenia. First we present the methodology that is currently is using for estimation of the yield curve dynamics in the Central bank of Armenia (hereafter CBA). Then we consider some alternative algorithms that would be useful (in terms of effectiveness and accuracy) for estimation and forecasting yield curve dynamics in Armenia.

The model used in this paper is the Diebold and Li (2006) variation of the Nelson-Siegel (1987) model. The variation of the famous Nelson-Siegel (1987) model has shown by many researchers to be the most accurate and best fit estimation of the yield curve. The attractiveness of factor models of the Nelson-Siegel type is due to its parsimony and good empirical performance. Models of this type can capture most of the behavior of the yield curve by using only three factors. That is why Nelson-Siegel three-factor models are the most popular amidst financial economists, investors, and central banks. Models with a larger number of factors were used by Svensson (1994), Almeida and Vicente (2008), Laurini and Hotta (2008) among others. Interpolation models were developed, for instance, by McCulloch (1971, 1975) and by Vasichek and Fong (1982).

According to Diebold and Li (2006) most researchers on the modeling of the term structure of interest rates have focused almost entirely on finding the best fit for a given time period, rather than the out of sample forecasting abilities of each model. However, having good yields forecasts is essential to calculate the market value of an asset portfolio, to assess fixed income derivatives, to build investment strategies and to develop appropriate monetary policy. Following this suggestion, for instance, Gasha et al., (2010) show that an estimated three-factor Nelson-Siegel model reproduces well the stylized facts of the US yield curve for the period of analysis, and provides a framework for assessing its dynamic evolution. Vicente and Tabak (2007) compared the Gaussian affine model with Diebold and Li model for Brazilian data and conclude that the latter model is slightly superior in terms of yield curve forecasts. Caldeira et al., (2010) differently from the literature on the Brazilian yield curve, they have put the Diebold-Li model in the State-space form and then the parameters are simultaneously estimated using Kalman filter. They show that the Kalman filter is the most suitable method for estimation of the model, generating better forecast for all maturities when we consider the forecasting horizons of one and three months.

This paper makse a dual contribution to the empirical estimation models of the Armenian yield curve dynamics: first we use the dynamic Diebold and Li model instead of static one (currently the static Nelson-Siegel model is using), which allows us to capture the main empirical facts behind the normal yield curve (that is upward sloping, concave and decreasing in interest rate volatility as maturities rise), and second we avoid the a prior selection of the parameters λ, that is we estimate the Diebold and Li dynamic model in a single step by using State-space Kalman filter approach. From the other side we use new data set on government bonds interest rates, which includes the period of global financial crisis. Our results could be of interest to practical macroeconomic policy-makers, especially from emerging and developing countries. The estimated parameters of the dynamic Diebold and Li model could be useful for calibration purposes in the process of developing other similar structural models. Also, our results could be of interest to practical econometricians who engage in model estimation and forecasting.

The paper is organized as follows. In the section 2 we explain the Nelson-Siegel static model and its variation by Diebold and Li (2006) as well as we discuss three widely used algorithms for estimation of yield curve dynamics, namely nonlinear optimization (static Nelson-Siegel model), Two-step procedure and the State-space Kalman filter model proposed by Diebold, Rudebush and Aruoba (2006). Throughout the paper we use the Diebold and Li (2006) variation of the three-factor Nelson-Siegel model, which interprets the factors as level, slope and curvature and thus allows us to make an out of sample forecast evaluation. Section 3 provides an overview of the Armenian bond market situation and important descriptive statistics of the yield curve dynamics between 2005M1 to 2017M12. Section 4 presents the estimation results achieved by using the above mentioned three algorithms as well as we presents the out of sample forecast evaluation results based on root mean squared forecasts error (RMSFE) criteria. The Diebold and Mariano (2005) test statistics is used to confirm if the differences among out of sample forecasts generated by the state-space Kalman filter approach are significantly differ comparing with another competing models.

2. METHODOLOGY

In this section we introduce the static latent factor model that Nelson and Siegel (1987) developed for the yield curve dynamics. Then we will introduce the dynamic model that was proposed by Diebold and Li (2006). For that first we will introduce the model by Diebold and Li (2006) and then we will go into the model in state-space form proposed by Diebold, Rudebush and Aruoba (2006). Now let's present each of mentioned method in turn.

Nelson and Siegel (1987) have shown that the yield curve dynamics can be fitted as a particular point of time by a linear combination of three smooth functions. The Nelson and Siegel model is given by:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{1}$$

As we can see from (1) the yield curve is modelled using three components. The first one remains constant when the term to maturity (τ) varies. The second factor has more impact on short maturities. The impact of the third factor increases with maturity, reaches a peak and then decays to zero. Thus, each one of these three factors governs the part of the yield curve, particularly long (β_1), short (β_2) and medium term (β_3). The last parameter λ governs the exponentially decay for (β_2) and (β_3).

Diebold and Li (2006) in their paper have introduced the dynamics into the original Nelson and Siegel model. As a result the Nelson and Siegel static model can be presented as dynamic model as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau} \right) \tag{2}$$

where β_{1t} is the long-term factor or level, β_{2t} is the short-term factor or slope and β_{3t} is the medium-term factor or curvature. The first component on β_{1t} takes value 1 (constant) and can be interpreted as overall level, which equally influence the short- and long-term rates. The second component on β_{2t} converges to one as $\tau \to 0$ and converges to zero as $\tau \to \infty$ for a given t. Hence this component mostly influence short-term interest rates.

The third component on β_{3t} converges to zero as $\tau \to 0$ and as $\tau \to \infty$, but is concave in τ for a given t. This component is therefore associated with medium-term interest rates. Finally the parameter λ_t governs the exponential decay rate; small values of λ_t produce slow decay and can better fit the curve at long maturities, while large values of λ_t produce fast decay and can better fit the curve at short maturities. Also λ_t governs where the component on β_{3t} achieves its maximum.

As we see from (2) λ introduces a non-linearity in the equation. Nelson and Siegel (1987) note that by fixing this parameter, the problem assumes a linear form and it can be estimated by simple regression using OLS. Thus in case we observe a series of interest rates $y_t(\tau_i)$ for a set of N different maturities $\tau_1 < \tau_2 < ... < \tau_N$ available at a given time t we can estimate the yield curve dynamics by the simple regression model.

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \epsilon_{it} \tag{3}$$

for $i = 1, ..., N$. The errors $\epsilon_{1t}, \epsilon_{2t}, ..., \epsilon_{Nt}$ are assumed to be independent with zero mean and constant variance σ_i^2 for a given t. The ordinary least squares algorithm can be used to estimate the β_{jt} coefficients, wheer $j = 1, 2, 3$. These estimates can be obtained for each t separately, because for a given time t we have different maturities and interest rates.

For example, based on this approach Diebold and Li (2006) have applied two step approach to forecast β_{jt} coefficients. According to this approach in the first step they fix λ parameter (λ_t = 0.0609 for all t) and then the β_{jt} coefficients are estimated by traditional least squares algorithm. In the second step using estimated in the first step β_{jt} coefficients they forecast β coefficients by using univariate (AR) or multivariate (VAR) algorithms (Koopman et al., (2007)). Thus, having forecasted values for β coefficients it is already possible to forecast the yield curve dynamics. Using this two-step approach Diebold and Li (2006) in their paper showed that the two-step forecasting approach does better than another alternative algorithms especially for the long maturities.

Diebold, Rudebush and Aruoba (2006) go a step further by recognizing that the Nelson-Siegel model can be represented as a state-space model when treating β_t as a latent factor.

In the case of VAR(1) the transition equation, which governs the dynamics of the state vector is defined by:

$$\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} = \begin{pmatrix} \mu_{\beta_1} \\ \mu_{\beta_2} \\ \mu_{\beta_3} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} \beta_{1t-1} \\ \beta_{2t-1} \\ \beta_{3t-1} \end{pmatrix} + \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{pmatrix} \tag{4}$$

The measurement equation which associates the interest rates of N maturities with the three unobserved components is given by:

$$\begin{pmatrix} y_t(\tau_t) \\ y_t(\tau_2) \\ . \\ . \\ . \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix} \times \begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ . \\ . \\ . \\ \epsilon_{Nt} \end{pmatrix} \tag{5}$$

The system comprising the measurment and transition equation can be written using matrix notation as follows:

$$\begin{aligned} y_t &= \Lambda(\lambda)\beta_t + \epsilon_t, & \epsilon_t &\sim NID(0, \Sigma_\epsilon) \\ \beta_{t+1} &= \mu + \Phi\beta_t + \eta_t, & \eta_t &\sim NID(0, \Sigma_\eta) \end{aligned} \tag{6}$$

from the above state-space model the 3×1 mean vector μ, the 3×3 autoregressive coefficent matrix Φ, the 3×3 variance matrix Σ_η and the $N \times N$ variance matrix Σ_ϵ are unknown and need to be estimated.

When the state-space form (6) is used , then two approaches can be employed to estimate the latent factors and the parameters. As it was mentioned above the initial approach to estimate model (6) was proposed by Diebold and Li (2006). This approach is based on the Two-step estimation procedure, which assumes that in the first step the measurement equation is estimated by using cross-sectional data, in which the estimates for the factors are obtained for each time period. In the second stage, the time dynamics of the parameters is specified and estimated as an AR(1) or VAR(1) process. Here the authors assume that the decay parameter is constant and equal $\lambda_t = 0.0609$.

Diebold, Rudebush and Aruoba (2006) showed that the model (6) can be estimated in a single step by using the State-space Kalman filter approach. Thus in the current paper we estimate model (6) with using both Two-step and State-space Kalman filter approaches. These two estimation algorithms we apply to Armenian government bonds yields data. We conduct out of sample forecasts experiments to see whether the State-space Kalman filter approach outperforms alternative models, like Random Walk and Two-step. To solve the model (6) we used MATLAB 2015a software, with the built-in Econometric Toolbox.

3. Data and descriptive statistics

The data used in this paper consists of monthly observations for all maturities of Armenian government zero-coupon bond yields (hereafter just government bond yields) from 2005M1 to 2017M12. There are 9 series of government bonds in Armenia with the following maturities: 3, 6, 9, 12, 24, 36, 48, 60, 120, 180 and 240 months. However due to extremely low levels of trading volumes, some missing data was present especially within the 180 and 240 maturities. To maximize the accuracy of the models applied in this paper, the missing data were imputed using algorithms that are based on the principal components analysis. Table 1 presents descriptive statistics for the Armenian yield curve dynamics.

As it was expected the mean yield to maturity increases with the maturity of the bond, which is in conjunction with the liquidity premium and the expectations hypothesis. As we can see the range for lower maturity is relatively high comparing with longer maturity. This can be explained by the significant jump in yields after the financial crisis of 2008 and by the negative spillover effects on the Armenian economy from the 2014 sanctions on Russian Federation. However starting from 2015 we have seen a steady decrease in the yields, nearing pre-crisis levels. The coefficient of variation (the ratio between standard

TABLE 1. Descriptive statistics for the actual yield curve dynamics

Maturity (Months)	Mean	Std.dev	Minimum	Maximum	Range	CV (%-th)
3	7.13	2.31	2.60	14.76	12.16	32.36
6	7.62	2.37	2.71	14.58	11.87	31.06
9	7.96	2.54	3.01	15.73	12.72	31.85
12	8.20	2.49	3.19	14.13	10.94	30.42
24	9.09	2.66	4.01	13.90	9.89	29.31
60	10.29	3.20	4.50	15.17	10.67	31.07
120	11.39	3.04	4.51	16.23	11.72	26.68
180	11.43	2.40	6.91	15.94	9.03	21.00
240	12.49	2.46	8.64	16.75	8.11	19.71

TABLE 2. Descriptive statistics for yield curve empirical factors

Factor	Mean	Std.dev	Minimum	Maximum	$\hat{\rho_1}$	$\hat{\rho_{12}}$	$\hat{\rho_{30}}$
Level	11.39	3.04	4.51	16.23	0.88	0.60	0.24
Slope	3.77	1.86	-2.06	7.52	0.58	0.36	-0.11
Curvature	-0.83	1.63	-5.34	6.12	0.12	0.04	-0.07

deviation and the mean in %-th term) measures the relative volatility over time for each maturity. The negative relationship between the coefficient of variation and maturities illustrates that for the given observation period bonds with shorter maturities have been relatively more volatile (32.36%) compared to bonds with longer maturities (19.71%).

Table 2 presents descriptive statistics for the Armenian yield curve empirical level, slope and curvature factors. First let's explain how we can calculate the descriptive statistics for empirical factors. First of all the empirical factors can be thought as proxies for the estimated level, slope and curvature. From the other side these proxies can be used as an good benchmark and cross-check for State-space Kalman filter estimates of the yield curve factors. Thus, the empirical factors are calculated as simple functions at different maturities: level = $y_t(3) + y_t(24) + y_t(120)/3$, slope = $y_t(3) - y_t(120)$, curvature= $2y_t(24) - y_t(3) - y_t(120)$ (Mumtaz and Surico (2009)). In the last three columns of Table 2 presented auto-correlation coefficients ($\hat{\rho_1}$) of lag 1, 12 and 30.

Thus, based a detailed look at the yield data we can identify the following stylized facts:

1. The average curve is positive. Thus, the longer the maturity of yields, the bigger is the average yield because the investors receives a premium for the undertaken risk;

2. The yield curve takes a variate of shapes along the period analyzed, particularly positive "hump-shaped", flatten (when the slope and curvature are close to 0) negative or U-shaped;

3. Yield's dynamics is persistent (the autocorrelation coefficient of $\hat{\rho_1}$ is 0.88), while spread's dynamics is less persistent ($\hat{\rho_k}$ of the slope is 0.58);

4. Long-term yields are less volatile than the short-term (the coefficient of variation is decreasing as the maturity increases - from 32.36% to 19.71%);

5. The level of the yield curve is highly persistent, but exhibits small variation relative to its mean;

6. The slope is less persistent than any single tield but highly variable relative to its mean;

7. The curvature is the least persistent of all factors and displays the largest variability relative to its mean.

4. Empirical results

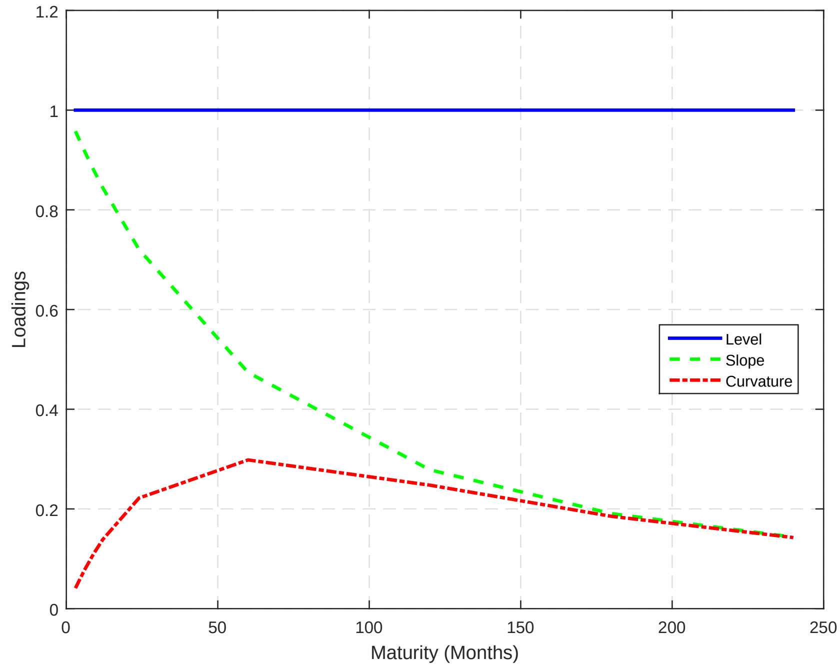
In the section 2 of the current paper the Diebold and Li model was put in the state-space form with a VAR(1) for the transition equation, which models the dynamics of the factors and a linear measurement equation that relates the observed yields to the state vector. As it was mentioned in section 3 in our paper we use Armenian yield curve actual data from 2005M1 to 2017M12 in monthly term. So in total we have 156 monthly observations for each of the 9 maturities. Unlike the Two-step method, in the State-space Kalman filter estimation the parameters are estimated in a single step. Relating with the decay parameter λ we can say that this parameter is estimated at the same time with other parameters and not determined a priori. But for Two-step estimation procedure first we should fix λ parameter. In other words, we intend to fix λ , so we can estimate the times series of level, slope and curvature. For example in the paper by Diebold and Li (2006) they fix λ at 0.0609 and then they estimate the time series of level, slope and curvature. Diebold, Rudebush and Aruoba (2006) estimate λ to be 0.077 and following this Yu and Zivot (2007) fix it at 0.077. We can not to take the value of λ as in the Diebold and Li (2006) paper because this value does not correctly characterized the dynamics of the Armenian yield curve data. Thus we have to calibrate the value of λ.

To calibrate λ we using the numerical optimization procedure. Based on the paper by Gilli et al. (2010) we set the following ranges for Nelson Siegel model parameters, particularly $0 < \beta_1 < 15$, $-15 < \beta_2 < 30$, $-30 < \beta_3 < 30$ and $0 < \lambda < 30$. Also we add additional constraints: $\beta_1 > 0$, $\beta_1 + \beta_2 > 0$ and $\lambda > 0$. Then the starting values for optimization procedure are randomly drawn from these ranges. We run the algorithm 500 times; for each restart we randomly choose a different starting values. We can run this algorithm for each month separately but it will be very time consuming procedures, because, in total we have 156 months and therefore we should run the algorithm 78000 times (500*156). Instead of running the algorithm 78000 times we have decided to run the algorithm 500 times for the average values of yield data calculated as simple arithmetical mean for the whole period under our consideration. Based on the numerical optimization results the obtained parameter of λ is equal to 0.0355. Using this value of λ we can calculate the approximate dynamics of the latent factors (level, slope and curvature factors). Figure 1 shows the dynamics of latent factors.

As we can see from Figure 1 the maximum value of curvature factor we can observe at 60 months (5 years). In other words the calibrated value of λ maximizes the curvature factor at maturity 60 months. For example in Diebold and Li (2006) the λ parameter is equal to 0.0609, which reaches its maximum in 30 months and 23.3 months for λ = 0.077. Thus, having calibrated λ parameter already we able to estimate dynamic Nelson-Siegel model by using Two-step and State-space Kalman filter models.

Let's start our analysis by comparing state transition matrix of the State-space Kalman filter model and the coefficients matrix obtained from Two-step model (Tables 3 and 4). First of all we can see that in both tables the diagonal elements are positive numbers and

FIGURE 1. Yield curve factor loadings



in most cases relatively large. This indicates that there exist persistent self-dynamics in each extracted factors. At the same time if we look on off-diagonal elements, then we see that in most cases the values are relatively small, which indicates the weak cross-factor dynamics.

TABLE 3. Estimated VAR(1) parameters for Two-step model

	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$
$\beta_{1,t}$	0.738***	0.268***	0.104***
	(0.050)	(0.060)	(0.017)
$\beta_{2,t}$	0.150***	0.570***	-0.090***
	(0.057)	(0.067)	(0.019)
$\beta_{3,t}$	0.860***	-0.070	0.627***
	(0.156)	(0.184)	(0.053)

Note: Standard errors in parenthesis. *, ** and *** indicates statistical significance at 10%, 5%, 1% significancy level respectively

Now let's compare the covariance matrices obtained from these two estimation algorithms. The results of estimated covariance matrices are presented in Table 5. First let's look at the diagonal elements of the covariance matrices. As we see the diagonal elements for both matrices are increasing as we proceed from the level to curvature factor.

This means that for these two approaches the curvature has relatively large variance comparing with level and slope factors variances. From the other side we also see that for Two-step model the three factors are characterizing with a significantly large variances comparing with the same variances in the case of State-space model, especially when we compare the variances of the curvature factor.

TABLE 4. Estimated transition matrix parameters for State-space model

	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$
$\beta_{1,t}$	0.847***	0.110***	0.033***
	(0.026)	(0.031)	(0.009)
$\beta_{2,t}$	0.198***	0.670***	-0.060***
	(0.036)	(0.043)	(0.012)
$\beta_{3,t}$	-0.001	0.555***	0.983***
	(0.055)	(0.065)	(0.019)

Note: Standard errors in parenthesis. *,** and *** indicates statistical significance at 10%, 5%, 1% significancy level respectively

TABLE 5. Estimated residuals covariance matrices for two competing models

Two-step model			State-space model		
1.400	-1.069	-2.697	0.383	-0.175	-0.285
-1.069	1.770	0.943	-0.175	0.728	-0.220
-2.697	0.943	13.328	-0.285	-0.220	1.671

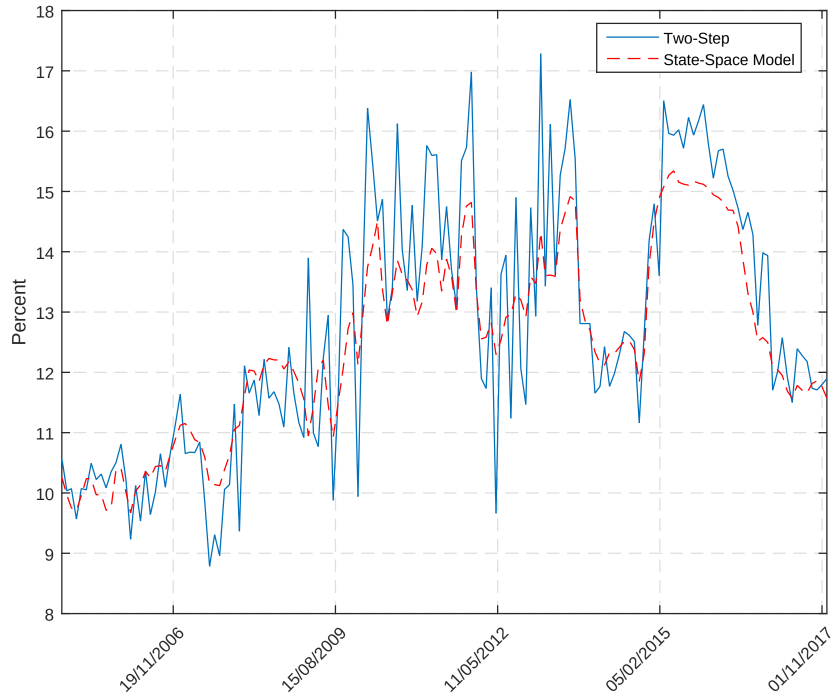
TABLE 6. Estimated factors average values

	Level	Slope	Curvature
Two-step model	12.712	-5.678	0.132
State-space model	12.066	-5.518	0.926

Now let's compare the average values of the extracted factors obtained from State-space model with those of the Two-step model. The results are presented in Table 6. From table we can see that in this case the estimated average values are in relatively close agreement for the level and slope factors, although the curvature differs between the two approaches.

The unobserved factors, which correspond to the level, slope and curvature factors of the Diebold and Li (2006) model are in primary focus, This is because from the dynamics of these factors are dependent the evolution of the yield curves. Now let us to analyze the

FIGURE 2. Level (Long-term Factor



dynamics of the unobserved factors inferred from each approach. In the Figures 2,3 and 4 are presented the dynamics of the level, slope and curvature factors obtained by using Two-step and State-space model (hereafter TS and SSM respectively).

To analyze the variability of the estimated factors (level, slope and curvature) we need to calculate the coefficients of variability. For that let us turn to the Table 6. From the table we see that the average values for the slope are negative for both estimation approaches. As it is known for negative data we can not calculate the standard coefficient of variation, because in the results we will obtain the negative values, which is not possible. From the other side we can calculate the variability by using the squared trend difference approach. Thus using this approach we have calculated the variability of the extracted factors and the results are presented in Table 7. As we can see from the Table the variability of the factors extracted by using TS model are larger than the corresponding values for the factors extracted by using SSM approach. The small sizes of variability means that SSM approach gives more smoothed factors than TS and the factors could be more forecast able.

FIGURE 3. Slope (Short-Term Factor)

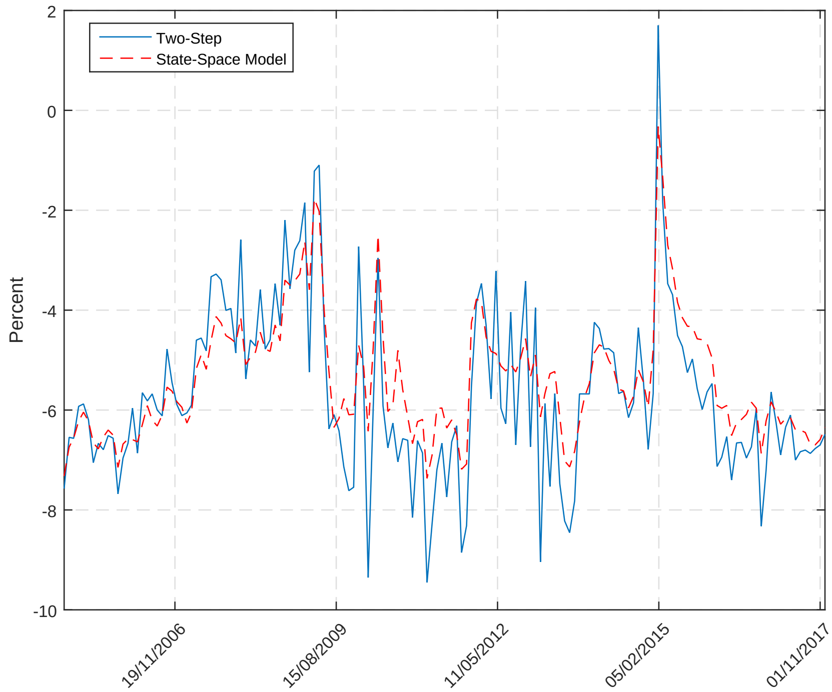
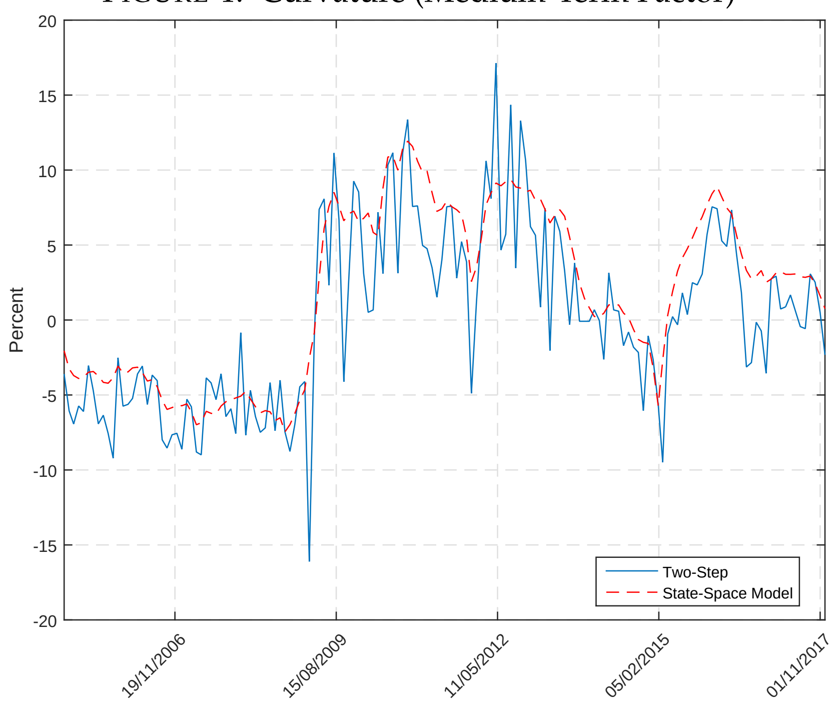


FIGURE 4. Curvature (Medium-Term Factor)

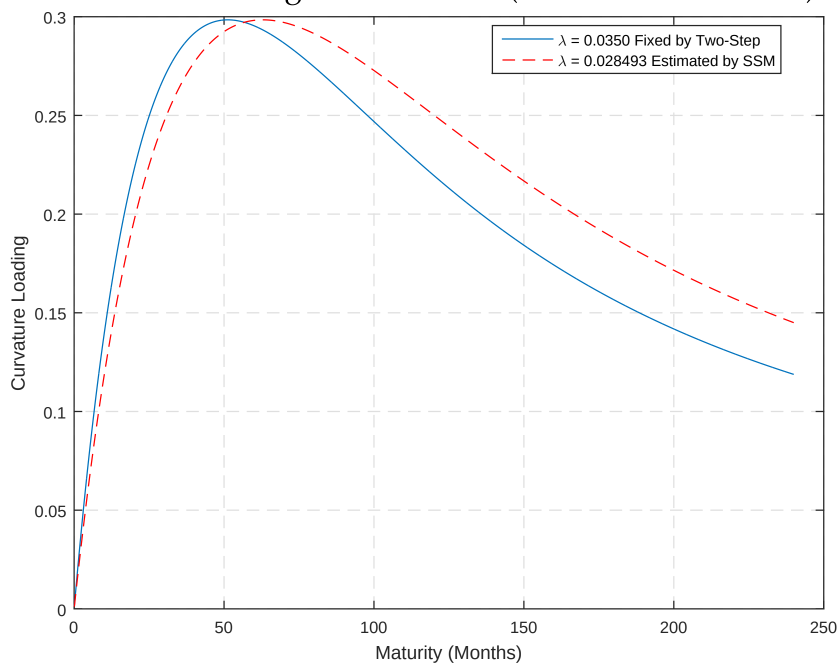


Now let's turn to the decay parameter λ that is associated with the curvature factor. The dynamics of calibrated and extracted decay parameter by the SSM approach is presented in the Figure 5. From the figure we see that the estimated by SSM decay parameter (0.0285) is somewhat lower than the decay parameter (0.0355) used in the TS model.

TABLE 7. The coefficients of variability in %-th

	Level	Slope	Curvature
Two-step model	1.33	1.43	3.84
State-space model	0.76	0.96	2.61

FIGURE 5. Loading on Curvature (Medium-term factor)



Thus, we conclude that the curvature factor according to the SSM reaches its highest level at about 70 months (about 6 years). From the other side although differences between two approaches exist, but the factors and decay parameters extracted from each approach are generally is in close agreement to each other. Therefore SSM approach that allows to estimate all unknown parameters simultaneously in a single step is more preferred.

Finally we can compare the performances of the two alternative models by using the out-of-sample forecast comparison procedure. The main idea of the out-of-sample comparisons is that we predict the three extracted factors separately by using TS and SSM approaches. Then, using predicted factors we calculate predicted values for the yield curve. To estimate the accuracy of the out-of-sample forecasts we compare predicted values with the actual values of the yield curves. To assess the goodness of fit we use root mean squared forecast error (RMSFE) index.

In the current paper in order to evaluate the out-of-sample forecast performances we use three competing models, particularly Random walk (hereafter RW), TS and SSM. Basically, in the RW model the next observation for yield is simply equal to the most recent observed value and an error term. In practice it is difficult to beat the RW model, in terms of out-of-sample forecasting accuracy. In this paper to conduct the out-of-sample forecast experiments we use recursive regression scheme, due to short data set. For our data the in-sample period is 2005M1-2014M9, (117 observations, or 75%-th of the whole sample), while the out of sample period is 2014M10- 2017M12 (39 observations, or 25%-th of the whole sample). The recursive simulation scheme proceeds as follows: First we estimate the models using sub sample 2005M1-2014M9 and generate one, three and six months ahead forecasts. Then we increase the sample size by one 2005M1-2014M10 and generate again one, three and six months ahead forecasts. By increasing sample size finally we will have 39 values for one, 37 values for three and 34 values for six months ahead forecast experiments. Next, we use the out-of-sample forecasts from recursive scheme to compute the corresponding root mean squared forecast errors (RMSFE). The results of calculations are presented in Table 8.

TABLE 8. RMSFE for out of sample forecasts (Nov 2014 to Dec 2017)

Maturity	One month ahead			Three months ahead			Six months ahead		
(Months)	RW	TS	SSM	RW	TS	SSM	RW	TS	SSM
3	0.978	0.998	0.938	2.014	1.197	1.081	2.697	1.138	1.184
6	0.994	0.989	0.952	1.885	1.106	1.049	2.432	0.850	0.928
9	1.137	1.131	1.127	1.991	1.228	1.219	2.395	0.753	0.876
12	0.845	0.825	0.860	1.758	0.900	0.943	2.306	0.650	0.757
24	0.768	0.725	0.843	1.572	0.709	0.895	2.436	0.521	0.788
60	0.730	0.625	0.784	1.492	0.619	0.787	2.480	0.621	0.760
120	0.698	0.682	0.677	1.280	0.748	0.758	2.094	0.766	0.618
180	0.891	0.751	0.596	1.293	0.598	0.607	2.050	0.540	0.465
240	0.752	0.813	0.804	1.140	0.785	0.902	1.938	0.691	0.795

Note: RW = Random Walk, TS = Two-step, SSM = State-space model

From Table 8 we see that RMSFE values calculated for TS and SSM are outperforms the corresponding values for RW model almost at all maturities and forecast horizons. When we comparing the RMSFE values for TS forecasts with the corresponding values for SSM we see that for some maturities and forecast horizons the SSM outperforms TS model and vice versa. Thus, we can conclude that the TS and SSM outperforms RW model almost at all maturities and forecast horizons, but when we comparing TS and SSM then we see that there is no definitive answer which one of the two competing models is better to use for forecasting. From the other side the question that could arises is whether these differences are statistically significant?

To confirm whether the differences among the out of sample forecasts generated by the above considered models are statistically significant, we applied the Diebold and Mariano (1995) test to compare forecasts. In this paper the Diebold-Mariano statistics values we calculate by regressing of the loss differentials on an intercept, using heteroscedastiocity autocorrelation robust (HAC) standard errors (Diebold, 2013). The loss differential between forecast 1 and 2 can be calculated as $l_t = (\epsilon_t^{ssm})^2 - (\epsilon_t^i)^2$, where ϵ_t^{ssm} is the forecast error from SSM model at time t,ϵ_t^i denote the forecast error from the alternative competing models $(i = RW, TS)$. Thus, regressing the loss differentials on an intercept using HAC standard errors we obtain the Diebold-Mariano statistics which are presented in the Table 9. The values presented in this table allow us to accept or reject the hypothesis that the two different models generate the same forecasts. If Diebold-Mariano statistics is larger (in absolute value) than some critical value (let say 1.96 at the 5% significancy level), then we reject the null hypothesis and conclude that the two models are different in the sense that they produce statistically different forecasts.

Now let's first compare the SSM and RW model. As we see from Table 9 for one step ahead forecast horizon there is not sufficient evidence to prefer SSM over the RW model, for three steps ahead forecast horizons the SSM significantly outperform the RW model, especially for bonds with longer maturities, for six steps ahead forecast horizons the SSM significantly outperform the RW model for all maturities. Thus, our results show that SSM significantly outperform RW model for longer forecast horizons. When we apply Diebold-Mariano test to SSM versus TS then we see that for all forecast horizons and maturities there is no strong evidence to prefer one model over the other.

TABLE 9. Estimated residuals covariance matrices for two approaches

Maturity	SS vs. TS		Three months ahead		Six months ahead	
(Months)	SS vs. TS	SS vs. RW	SS vs. TS	SS vs. RW	SS vs. TS	SS vs. RW
3	-0.43	-0.35	-0.55	-1.33	0.32	-2.00**
6	-0.31	-0.34	-0.36	-1.35	0.58	-1.95*
9	-0.02	-0.09	-0.08	-1.30	1.14	-1.88*
12	0.72	0.23	0.66	-1.28	1.34	-1.74*
24	1.81*	1.00	2.33**	-1.73*	2.27**	-1.99**
60	3.08***	0.98	2.36**	-2.09**	2.40**	-2.45**
120	0.20	-0.26	0.15	-1.94*	-1.88*	-2.20**
180	-0.71	-3.77***	0.22	-2.75***	-0.80	-2.85***
240	-0.15	0.65	1.55	-1.09	1.33	-1.78*

5. CONCLUSIONS

In this paper, the Diebold and Li (2006) model has been applied for actual data on the Armenian yield curve. Also, we put the Two-step method in the State-space form and then simultaneously estimate all unknown parameters of the Nelson-Siegel model by the State space model, using Kalman filter. The maximum likelihood allows for the joint estimation of all parameters of the model, preventing the a prior selection of the decay parameter. This is one important advantage of the SSM approach, when compared with the Two-steps approach. When we conduct out-of-sample forecast evaluation we compare SSM with two alternative models, namely Random walk and Two-steps. The results show that the forecasts based on the SSM comparing with RW forecasts are better almost for all maturities and forecasting horizons. From the other side when we compare SSM with TS model, then we see that SSM outperform TS only in some cases and there is no definitive winners. We also apply Diebold-Mariano test statistics to see whether forecasts produced by alternative models are statistically different. When we compare SSM with RW model we see that for one step ahead forecast horizon there is not sufficent evidence to prefer SSM over the RW, for three months forecast horizons the SSM model significantly outperform the RW for longer maturities, while for six-months forecast horizons

SSM significantly outperform the RW at all maturities. When we apply the Diebold-Mariano test to SSM versus TS, then we conclude that there is no sufficient evidence to prefer one model over another and both models can be successfully applied.

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